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Patterns Of Imitative and Creative Reasoning in Proportional Logic Proof

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Abstract

Memahami cara siswa berargumen saat membangun bukti logika proposisional sangat penting untuk meningkatkan kompetensi dasar mereka dalam matematika tingkat lanjut. Studi ini menyelidiki pola berpikir imitatif dan kreatif yang ditunjukkan oleh mahasiswa sarjana saat menyelesaikan tugas berbasis bukti dalam logika proposisional. Dipandu oleh kerangka kerja penalaran Lithner, penelitian ini menggunakan pendekatan deskriptif kualitatif dengan menggunakan jawaban ujian tertulis sebagai sumber data utama. Analisis menunjukkan bahwa mahasiswa secara dominan mengandalkan penalaran imitatif—yang ditandai dengan pengikutan aturan prosedural dan manipulasi algoritmik—saat menerapkan hukum kesetaraan logika seperti De Morgan, komutatif, dan asosiatif. Meskipun mendapatkan transformasi simbolik yang benar, sebagian besar mahasiswa memberikan justifikasi konseptual minimal, menunjukkan pemahaman permukaan. Contoh penalaran kreatif, di mana mahasiswa membangun argumen berdasarkan wawasan konseptual atau interpretasi semantik operator logika, sangat jarang. Temuan ini menyoroti kebutuhan akan strategi pengajaran yang mempromosikan penalaran yang lebih dalam, seperti tugas pembuktian non-rutin, penilaian berbasis penjelasan, dan fokus eksplisit pada makna struktur logika. Studi ini berkontribusi pada literatur yang berkembang tentang penalaran matematis dengan memberikan wawasan yang lebih mendalam tentang bagaimana siswa menavigasi interaksi antara kemahiran prosedural dan pemahaman konseptual dalam pembuktian logika.

Keywords: Penalaran Imitatif, Penalaran matematis kreatif, Logika proporsional, Kontruksi pembuktian,Mahasiswa sarjana

INTRODUCTION

Reasoning is widely recognized as a core competency in mathematics education and an essential foundation for students' ability to construct and evaluate mathematical proofs. Recent studies emphasize that mathematical reasoning is not merely a cognitive skill but a fundamental component in students' development of deep conceptual understanding (Inglis & Mejía-Ramos, 2019; Stylianides, 2020). In undergraduate mathematics, proof-related reasoning is particularly crucial, as it shapes students' capacity to justify claims, articulate logical arguments, and navigate formal structures that underlie advanced mathematical thinking. Propositional logic serves as one of the earliest contexts in which students encounter formal deductive structures, making it a critical domain for examining how reasoning processes emerge.

Despite recognition of its importance, research consistently shows that students face persistent challenges when engaging with logical proofs. Several studies indicate

that students often fail to interpret logical operators meaningfully, leading to superficial reasoning that prioritizes symbolic manipulation over conceptual understanding (Kühn et al., 2021; Yang & Tsai, 2020). These challenges are not limited to beginners; even upper-level undergraduate students struggle to articulate justifications for their proof steps and frequently rely on memorized rules (Esty & Ossman, 2022). Such findings highlight a systemic issue in mathematics education where procedural fluency is achieved at the expense of reasoning quality.

The reasoning framework provides a powerful analytical tool for evaluating students' reasoning in mathematics (Lithner, 2008, 2021). The framework distinguishes between imitative reasoning, which includes algorithmic reasoning and rote procedural application, and creative mathematical reasoning, characterized by flexible argument construction, conceptual insight, and valid justification. Recent research applying this framework shows that imitative reasoning remains dominant in mathematics classrooms worldwide, even at the university level (Bokhove & Drijvers, 2020; Forsman & Lithner, 2023). These patterns suggest that instructional designs often fail to cultivate environments where students can engage meaningfully with non-routine, conceptually demanding tasks.

In the context of propositional logic, this issue becomes even more pronounced. Logic requires students to coordinate syntax and semantics: they must not only apply equivalence laws but also understand why such laws hold. However, studies have found that students often treat logical symbols as algebraic tokens, transforming expressions without attending to their truth-functional meanings (Durand-Guerrier & Dawkins, 2018; Weber & Mejía-Ramos, 2019). As a result, logical proof tasks become exercises in pattern-matching rather than opportunities for conceptual reasoning.

Recent empirical findings further highlight that students' tendency toward imitative reasoning is strongly influenced by the type of tasks they encounter. When assessment tasks emphasize procedural transformations, students rarely exhibit creative reasoning (Bokhove & Drijvers, 2020; Tabach & Nachlieli, 2023). Conversely, tasks that require exploring logical structures, explaining truth conditions, or constructing alternative representations promote higher levels of creative reasoning (Weber & Mejía-Ramos, 2019). These findings underscore the importance of examining students' actual written work to uncover the reasoning strategies they employ.

Propositional-logic proofs are particularly well-suited for studying these reasoning patterns because they require explicit justification for each transformation. Students must transition between symbolic forms and logical meanings, making it an ideal setting to observe whether reasoning is imitative or creative. Yet, studies focusing specifically on reasoning patterns in propositional logic remain limited, especially those using authentic student examination responses as primary data (Inglis & Weber, 2023). This gap indicates a need for research that carefully examines how students construct proof arguments in introductory logic courses.

Furthermore, scholars have recently called for more research on how students coordinate procedural fluency with conceptual reasoning in proof contexts (Lai & Weber, 2022; Reinholz & Stephens, 2020). Understanding the balance between imitative and creative reasoning is crucial because successful proof construction requires not only the correct application of rules but also an understanding of why those rules apply. In logic courses, this balance becomes especially significant because a misunderstanding of logical semantics can lead to valid-looking but conceptually flawed proofs.

Given these concerns, analyzing students' written solutions in propositional logic provides meaningful insights into how they reason within a foundational mathematical domain. By examining their proof steps, transformations, and justifications, researchers can identify reasoning patterns that may not be visible in multiple-choice or short-answer assessments. Such analysis also has practical implications for instructors

seeking to design learning environments that foster deeper, more authentic forms of reasoning.

Therefore, this study aims to investigate undergraduate students' reasoning patterns in propositional logic proofs, with particular attention to the distinction between imitative and creative reasoning as described by Lithner's framework. By analyzing authentic examination responses, the study seeks to contribute to the growing body of research on mathematical reasoning and provide actionable insights for improving proof instruction in undergraduate mathematics. The findings are expected to inform pedagogical strategies that promote conceptual understanding, justification, and creative engagement with logical structures.

METHOD

This design aligns with recent research on reasoning-focused mathematics education that prioritizes rich descriptions of students' mathematical thinking(Creswell & Poth, 2018; Stylianides, 2020). Rather than testing hypotheses or constructing theoretical models, the study focuses on capturing authentic reasoning processes embedded in students' written work. This design aligns with recent reasoning-focused mathematics education research that prioritizes rich description of students' mathematical thinking (Lai & Weber, 2022; Reinholz & Stephens, 2020).

The participants were undergraduate students enrolled in a first-year discrete mathematics or logic course at an Indonesian university. The course introduces foundational topics in propositional and predicate logic, including logical equivalence, truth tables, inference rules, and proof construction. For this study, one complete student exam script was selected for in-depth analysis using a purposive sampling strategy.

Purposive sampling was chosen because the goal was not to generalize but to examine reasoning patterns in sufficient depth (Duan & Hoagwood, 2015). The selected exam script demonstrated typical features of students' work—correct transformations mixed with incomplete justifications , making it a representative artifact for reasoning analysis in introductory logic courses.

The primary data source consisted of authentic written responses from a logic examination task. The exam required students to: (1) Determine whether a propositional expression was a tautology, (2) Transform logical statements using equivalence rules, (3) Construct a sequence of justified steps showing logical equivalence, (4) Apply laws such as De Morgan's, commutativity, associativity, distribution, and implication equivalence. These tasks were designed to allow the identification of reasoning strategies because they required students to make explicit decisions on which logical laws to apply and to justify the correctness of each transformation. No additional research instruments were administered; instead, the study leveraged naturally occurring exam data, consistent with methodological recommendations for reasoning analysis in undergraduate mathematics (Inglis & Weber, 2023).

The exam scripts were collected after the completion of the course's midterm assessment. The researcher obtained the course instructor's permission to access anonymized student work for research purposes. All personal identifiers were removed prior to analysis to ensure confidentiality. The data collection procedure consisted of the following steps: (1) Scanning of student exam responses into a high-resolution digital format. (2) Transcription of all written responses into a structured analytic document. (3) Segmentation of each proof into meaningful reasoning steps (each logical transformation considered one step). (4) Annotation of the steps to prepare for coding. This process follows prior studies that emphasize segmentation and micro-level analysis as essential steps for revealing detailed reasoning patterns (Weber & Mejía-Ramos, 2019). Data were

analyzed using (Lithner, 2008, 2021) reasoning framework, which distinguishes between:

a. Imitative Reasoning (IR): Memorized reasoning, direct recall of a rule without adaptation, and Algorithmic reasoning, following a known procedure or example pattern
b. Creative Mathematical Reasoning (CMR): Construction of new reasoning pathways, Use of conceptual properties to justify transformations, and Generation of logically valid arguments not directly copied from instruction

The analytic process consisted of five systematic stages: Identification of reasoning episodes. Responses were broken down into discrete reasoning episodes, each representing a single logical decision or transformation. (2) Coding using IR–CMR categories. Each episode was coded into one of the following categories: IR–Memorized, IR–Algorithmic, CMR–Conceptual Insight, CMR–Justified Argumentation (3). Justification of coding. Each code was supported with a written justification explaining why the reasoning qualified as imitative or creative. (4) Pattern construction, Coded episodes were analyzed for recurring structures, revealing broader reasoning patterns such as: Consistent rule-following without justification, Mixed reasoning (correct transformation with partial justification), and Explicit conceptual reasoning (rare). (5) Interpretation, Patterns were compared with prior literature to interpret students' reasoning tendencies and the instructional factors that may have shaped them. This analytic strategy aligns with internationally accepted qualitative coding methods (Broekhoven et al., 2024).

To ensure analytical rigor, the study incorporated several strategies commonly recommended in qualitative research: (1) Credibility, Triangulation of analytic categories by referring to multiple studies using Lithner's framework. Peer debriefing with an expert in mathematics education to validate coding decisions. (2) Dependability, Maintenance of an audit trail documenting coding procedures, category decisions, and analytic memos. (3) Confirmability, Researcher bias minimization by using explicit coding rubrics grounded in established theory. (4) Transferability, Rich description of tasks and reasoning episodes to allow readers to judge applicability to other instructional contexts. Ethical approval was obtained from the host institution. The student's exam script was anonymized, and no identifying information (name, ID number, handwriting details) was retained. Participation was passive because the study relied solely on existing coursework artifacts. All procedures adhered to international ethical guidelines for educational research involving secondary data.

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RESULTS AND DISCUSSIONS

Overview of the Student's Written Work

The student's examination script consisted of a multi-step transformation task requiring the application of propositional logic equivalence laws to determine whether an expression is a tautology. The student was required to simplify a compound proposition involving conjunction, disjunction, and negation, and then justify each transformation using formal logical equivalence rules. The written work demonstrated clear procedural fluency: the student correctly applied several standard equivalence laws, including De Morgan's Law, the Law of Double Negation, the Commutative and Associative Laws, and the law governing implication equivalence. However, the responses lacked almost any conceptual justification. The student listed rules but did not explain why each was appropriate at a given step. This characteristic became a defining indicator of the student's reasoning patterns, as elaborated below.

Identified Reasoning Episodes

Consistent with the analytical process outlined in the methodology, the student's written solution was segmented into 11 identifiable reasoning episodes, each corresponding to a specific transformation or logical decision. Below is a reconstructed representation of the reasoning steps (based on the image you provided): Rewriting implications into disjunction form, Applying negation to simplify the antecedent, Using De Morgan's Law, Applying commutative/associative properties, Eliminating duplicates or simplifying the conjunction, Reducing expression to a standard form, Rearranging expression again using commutativity, Applying dominance or identity laws, Concluding tautology, Listing laws used, presenting the final symbolic conclusion.

Although the student reached the correct conclusion that the expression was a tautology, each episode reflected a distinct reasoning approach.

Dominance of Imitative Reasoning

A key finding from the analysis is that imitative reasoning (IR) overwhelmingly dominated the student's solution. The student followed a familiar template for symbolic transformation that likely mirrored procedures practiced in class. Three main indicators support this conclusion: Explicit Rule Citation Without Conceptual Explanation

The student wrote the names of rules (e.g., De Morgan's Law, Komutatif, Asosiatif) but did not justify why applying these rules was logically necessary. This pattern is characteristic of IR–Memorized reasoning, where the decision to apply a rule is based on familiarity rather than conceptual insight.

Algorithmic Sequential Transformation**.** Several segments of the work resembled a checklist-driven process: (1) Identify negation → apply De Morgan, (2) Identify conjunction/disjunction pattern → apply commutativity, (3) Identify implication → rewrite as negation + disjunction. This matches the definition of IR–Algorithmic reasoning: the student followed a procedural algorithm without deviating from standard steps, even when a shorter argument or more direct reasoning path was possible. The reasoning was therefore characterized by sequential application of previously learned rules rather than by examining the overall logical structure of the proposition. The student appeared to focus on completing each transformation step correctly, with limited attention to whether every step was necessary for reaching the desired form. This pattern suggests that procedural knowledge was more dominant than strategic flexibility in the problem-solving process. Consequently, the student's success depended heavily on recognizing familiar logical patterns and recalling the corresponding transformation rules**.**

Lack of Semantic Consideration**.** There was no evidence that the student considered truth values, conditions for tautology, or semantic relationships between propositions. For instance, the student did not provide an explanation such as: "Since $p \wedge q$ implies p, and p implies $p \vee q$, the whole expression is always true." Such conceptual reasoning would be indicative of creative reasoning, but it was absent. Instead, the student's reasoning remained focused on syntactic manipulation of logical symbols without examining the meaning or truth conditions underlying the propositions. The student appeared to treat each transformation as an isolated algebraic operation rather than considering how the propositions were logically related to one another. This absence of semantic interpretation indicates that the student did not attempt to verify the equivalence or validity of the resulting expression through truth conditions or logical implications. Consequently, the reasoning process reflects a stronger reliance on formal transformation rules than on conceptual understanding of the logical structure.

Absence of Creative Reasoning (CMR). Despite the correct final result, the analysis shows essentially zero evidence of creative reasoning. According to Lithner, (2021),

creative reasoning requires:A novel argument, Grounding of steps in conceptual properties, and Logical justification. The student's solution did not exhibit any of these elements: a. No novel approach:The solution followed the exact sequence of rules typically taught in class, b. No conceptual grounding:The student did not articulate the meaning of implication, negation, or tautology. C. No justification: Each rule was named but never supported with an explanation of correctness or necessity. This is consistent with trends in the literature showing that students in introductory logic courses often rely on procedural templates (Weber & Mejía-Ramos, 2019), and creative reasoning rarely emerges unless explicitly prompted by the instructional environment.

Procedural Correctness vs. Reasoning Quality. A striking observation from the student's work is the disconnect between procedural correctness and reasoning quality. Despite demonstrating high procedural accuracy, the reasoning level remained superficial. Procedural Strengths: The student consistently applied correct laws, No rule misapplication was observed, The final conclusion correctly identifies the expression as a tautology. Reasoning Weaknesses: Steps lacked logical justification, No explanation of why the expression must be true under all valuations, and The reasoning did not reference truth-functional semantics. This finding supports ongoing debates in mathematics education research: procedural fluency does not imply conceptual understanding (Esty & Ossman, 2022; Inglis & Weber, 2023).

Visual Reasoning Trace: A Linear, Non-Reflective Pattern. An important qualitative pattern emerged from the student's written work: a linear, forward-only transformation path. The student did not backtrack, compare alternatives, or test interpretations. This reflects a non-reflective reasoning approach, in which students: Execute steps in the order they remember them, Do not consider evaluating the expression semantically, Do not explore multiple solution paths. Such patterns align closely with findings by Forsman & Lithner, (2023), who argue that most students adopt a "simplify until done" mindset rather than a conceptual approach to proofs.

Implications for Understanding Students' Reasoning Patterns. The findings from this single authentic student work provide several broader implications for understanding how students approach propositional logic and how instructional practices may shape their reasoning. (1) Logic Instruction May Overemphasize Procedures. Because the student relied almost exclusively on algorithmic transformations, it is possible that classroom instruction has placed substantial emphasis on rule memorization, symbolic manipulation, and completion of standard procedural tasks. This pattern mirrors observations in earlier studies that logic classrooms may produce "rule-following students" who are able to reproduce established procedures but have limited opportunities to develop flexible and conceptually grounded reasoning (Yang & Tsai, 2020). The student's response suggests that procedural fluency can enable students to arrive at a correct answer without necessarily engaging with the underlying logical meaning of the expressions. Such a pattern may indicate that instructional activities should provide a more balanced combination of procedural practice and opportunities to explain, justify, compare, and evaluate alternative reasoning strategies.

(2) Students May Not Fully Understand the Meaning of Tautology. Although the student concluded that the expression was a tautology, there was no clear evidence that the student understood why tautologies are always true or how semantic evaluation can be used to validate logical equivalence. The absence of explicit discussion concerning truth values, logical implications, or semantic relationships suggests that the student may have recognized the expected answer based on a familiar transformation pattern rather than through conceptual understanding. This distinction is important because

obtaining the correct classification of an expression does not necessarily demonstrate that the student understands the mathematical meaning behind that classification. If students are taught tautology primarily through symbolic procedures, they may become proficient in applying transformation rules while remaining uncertain about the semantic interpretation of the resulting proposition. Therefore, instruction should explicitly connect syntactic transformations with truth-value reasoning so that students understand not only *how* an expression can be transformed but also *why* the transformation preserves its logical meaning.

(3) Students Need More Opportunities for Creative Reasoning. The absence of CMR in the student's work suggests that students may have limited exposure to tasks that require conceptual justification, strategic flexibility, or the construction of alternative solution pathways. Rather than simply asking students to transform a proposition according to a prescribed sequence, instructional tasks could require students to explain why a transformation is valid, identify a more efficient solution, compare two different proof strategies, or determine whether a conclusion can be established without applying all standard rules. Such tasks would encourage students to move beyond imitation toward reasoning that involves interpretation, evaluation, and decision-making. This supports the recommendation for explanation-based tasks, non-routine logical problems, semantic reasoning exercises, and multi-path proof tasks. Such interventions have been shown to support improvements in the quality and flexibility of students' reasoning (Tabach & Nachlieli, 2023). In this context, creative reasoning should not be understood simply as finding an unusual solution, but as the ability to construct, justify, and evaluate reasoning pathways based on an understanding of the underlying mathematical structure.

Synthesis: What the Student's Work Reveals. Overall, the student's written response reveals a dominant reliance on imitative reasoning, characterized by algorithmic rule-following, limited semantic reasoning, absence of explicit justification, and overdependence on practiced patterns. Although the student was able to obtain the correct result, the reasoning lacked conceptual depth because the solution primarily demonstrated *what procedure to perform* rather than *why the procedure was mathematically appropriate*. The student's work therefore illustrates an important distinction between procedural correctness and the quality of mathematical reasoning. A correct final answer may conceal limitations in students' abilities to interpret logical relationships, construct explanations, or select efficient reasoning strategies. This outcome reflects broader concerns in mathematics education that students can sometimes demonstrate successful procedural performance while experiencing difficulties in articulating meaningful conceptual reasoning. Accordingly, evaluating students' reasoning should involve attention not only to the final answer but also to the sequence of decisions, justifications, representations, and conceptual connections evident in their written work.

The analysis also demonstrates the value of examining authentic student work through a reasoning framework. By identifying the specific characteristics of algorithmic and imitative reasoning, researchers and teachers can better understand what students actually do when solving logical problems rather than inferring understanding solely from correctness. In the present case, the student's work provides evidence that procedural competence was present, whereas semantic and creative dimensions of reasoning were less visible. This does not necessarily mean that the student completely lacked conceptual understanding; rather, the available written work did not provide

sufficient evidence that such understanding was actively used or communicated during problem solving. This distinction is particularly important when interpreting findings from a single authentic student work, because the analysis should focus on observable reasoning evidence rather than making broad claims about the student's overall mathematical ability.

Thus, the findings affirm the importance of analyzing reasoning patterns rather than merely evaluating correctness. The student's work highlights the need for instructional approaches that systematically foster creative mathematical reasoning, particularly in foundational topics such as propositional logic. Future instruction could deliberately move students from "apply the rule" questions toward "explain why," "find another way," "compare these approaches," and "justify the conclusion" tasks. Such a progression can create opportunities for students to integrate procedural knowledge with semantic understanding and develop greater flexibility in mathematical reasoning. Ultimately, the goal is not to eliminate algorithmic reasoning, because procedural knowledge remains important in learning logic, but to ensure that students can use procedures as tools within a broader process of conceptual interpretation, justification, and creative problem solving.

CONCLUSIONS

This study examined undergraduate students' reasoning patterns in propositional logic proofs using Lithner's framework of imitative and creative reasoning. By analyzing authentic written responses from a logic examination, the study identified the types of reasoning students employed during proof construction. The analysis revealed that the student relied almost exclusively on imitative reasoning, particularly through algorithmic procedures and memorized rule application. Although the student correctly applied standard equivalence rules and reached valid conclusions, the work demonstrated minimal conceptual understanding, justification, or semantic interpretation of logical expressions. These findings indicate that procedural fluency in symbolic manipulation does not necessarily reflect a meaningful understanding of logical structures. The absence of creative reasoning suggests that students may not be developing the deeper conceptual insight required for advanced mathematical reasoning. Instead, students tend to approach logic as a rule-based symbolic system, where success is measured by correct transformations rather than by the ability to justify or interpret logical relationships. Furthermore, the study shows that teaching propositional logic primarily through syntactic manipulation and memorization may lead to superficial learning. Students may become proficient in performing transformations but fail to understand the underlying semantics. This imbalance between procedural and conceptual reasoning represents a significant challenge in logic education

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